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What is a bilinear form.
Bilinear list.
Bilinear form example.

Scalar-valued bilinear function
In mathematics, a bilinear form on a vector space *V* (the elements of which are called vectors) over a field *K* (the elements of which are called scalars) is a bilinear map *V* × *V* → *K*. In other words, a bilinear form is a function *B* : *V* × *V* → *K* that is linear in each argument separately: *B*(*u* + *v*, *w*) = *B*(*u*, *w*) + *B*(*v*, *w*) and *B*(*u*, *v*) = λ*B*(*u*, *v*) *B*(*u*, *v* + *w*) = *B*(*u*, *v*) + *B*(*u*, *w*) and *B*(*u*, λ*v*) = λ*B*(*u*, *v*)
The dot product on 




R

n




{\displaystyle \mathbb {R} ^{n}}

 is an example of a bilinear form.[1] The definition of a bilinear form can be extended to include modules over a ring, with linear maps replaced by module homomorphisms. When *K* is the field of complex numbers *C*, one is often more interested in sesquilinear forms, which are similar to bilinear forms but are conjugate linear in one argument.
Coordinate representation
Let *V* ≅ 




K

n




{\displaystyle B(\mathbf {v} )=\mathbf {x} ^{\textsf {T}}A\mathbf {y} =\sum \_{i,j=1}^{n}x\_{i}A\_{ij}y\_{j}.}

 A bilinear form has different matrices on different bases. However, the matrices of a bilinear form on different bases are all congruent. More precisely, if {*f* 1, ..., *f* *n*} is another basis of *V*, then 




f

j




=

∑

i
=
1


n




S

i
j




e

i


,


{\displaystyle \mathbf {f} \_{j}=\sum \_{i=1}^{n}S\_{ij}\mathbf {e} \_{i},}

 where the 




S

i
j




{\displaystyle S\_{ij}}

 form an invertible matrix *S*. Then, the matrix of the bilinear form on the new basis is *STAS*.
Maps to the dual space
Every bilinear form *B* on *V* defines a pair of linear maps from *V* to its dual space *V*\*. Define *B* 1, *B* 2: *V* → *V*\* by *B* 1(*v*)(*w*) = *B*(*v*, *w*) *B* 2(*v*)(*w*) = *B*(*w*, *v*) This is often denoted as *B* 1(*v*) = *B*(*v*, ·) *B* 2(*v*) = *B*(·, *v*) where the dot (·) indicates the slot into which the argument for the resulting linear functional is to be placed (see Currying). For a finite-dimensional vector space *V*, if either of *B* 1 or *B* 2 is an isomorphism, then both are, and the bilinear form *B* is said to be nondegenerate. More concretely, for a finite-dimensional vector space, non-degenerate means that every non-zero element pairs non-trivially with some other element: 



B
(
x
,
y
)
=
0


{\displaystyle B(x,y)=0}

 for all 



y
∈
V


{\displaystyle y\in V}

 implies that 



x
=
0


{\displaystyle x=0}

 and 



B
(
x
,
y
)
=
0


{\displaystyle B(x,y)=0}

 for all 



x
∈
V


{\displaystyle x\in V}

 implies that 



y
=
0


{\displaystyle y=0}

. The corresponding notion for a module over a commutative ring is that a bilinear form is unimodular if *V* → *V*\* is an isomorphism. Given a finitely generated module over a commutative ring, the pairing may be injective (hence "nondegenerate" in the above sense) but not unimodular. For example, over the integers, the pairing *B*(*x*, *y*) = 2*xy* is nondegenerate but not unimodular, as the induced map from *V* = *Z* to *V*\* = *Z* is multiplication by 2. If *V* is finite-dimensional then one can identify *V* with its double dual *V*\*\* . One can then show that *B* 2 is the transpose of the linear map *B* 1 (if *V* is infinite-dimensional then *B* 2 is the transpose of *B* 1 restricted to the image of *V* in *V*\*\*). Given *B* one can define the transpose of *B* to be the bilinear form given by *t**B*(*v*, *w*) = *B*(*w*, *v*). The left radical and right radical of the form *B* are the kernels of *B* 1 and *B* 2 respectively;[2] they are the vectors orthogonal to the whole space on the left and on the right.[3] If *V* is finite-dimensional then the rank of *B* 1 is equal to the rank of *B* 2. If this number is equal to dim(*V*) then *B* 1 and *B* 2 are linear isomorphisms from *V* to *V*\*. In this case *B* is nondegenerate. By the rank-nullity theorem, this is equivalent to the condition that the left and equivalently right radicals be trivial. For finite-dimensional spaces, this is often taken as the definition of nondegeneracy.
Definition: *B* is nondegenerate if *B*(*v*, *w*) = 0 for all *w* implies *v* = 0. Given any linear map *A*: *V* → *V*\* one can obtain a bilinear form *B* on *V* via *B*(*v*, *w*) = *A*(*v*)(*w*). This form will be nondegenerate if and only if *A* is an isomorphism. If *V* is finite-dimensional then, relative to some basis for *V*, a bilinear form is degenerate if and only if the determinant of the associated matrix is zero. Likewise, a nondegenerate form is one for which the determinant of the associated matrix is non-zero (the matrix is non-singular). These statements are independent of the chosen basis. For a module over a commutative ring, a unimodular form is one for which the determinant of the associate matrix is a unit (for example 1), hence the term; note that a form whose matrix determinant is non-zero but not a unit will be nondegenerate but not unimodular, for example *B*(*x*, *y*) = 2*xy* over the integers. Symmetric, skew-symmetric and alternating forms
We define a bilinear form to be symmetric if *B*(*v*, *w*) = *B*(*w*, *v*) for all *v*, *w* in *V*; alternating if *B*(*v*, *v*) = 0 for all *v* in *V*; skew-symmetric or antisymmetric if *B*(*v*, *v*) = −*B*(*w*, *v*) for all *v*, *w* in *V*; Proposition Every alternating form is skew-symmetric. Proof This can be seen by expanding *B*(*v* + *w*, *v* + *w*). If the characteristic of *K* is not 2 then the converse is also true: every skew-symmetric form is alternating. If, however, char(*K*) = 2 then a skew-symmetric form is the same as a symmetric form and there exist symmetric/skew-symmetric forms that are not alternating. A bilinear form is symmetric (respectively skew-symmetric) if and only if its coordinate matrix (relative to any basis) is symmetric (respectively skew-symmetric). A bilinear form is alternating if and only if its coordinate matrix is skew-symmetric and the diagonal entries are all zero (which follows from skew-symmetry when char(*K*) ≠ 2). A bilinear form is symmetric if and only if the maps *B* 1, *B* 2: *V* → *V*\* are equal, and skew-symmetric if and only if they are negatives of one another. If char(*K*) ≠ 2 then one can decompose a bilinear form into a symmetric and a skew-symmetric part as follows 




B

+


=



1

2



(
B
+
t
B
)


B

−


=



1

2



(
B
−
t
B
)


,


{\displaystyle B^{+}={\frac {1}{2}}(B+{\text{t}}B)\qquad B^{-}={\frac {1}{2}}(B-{\text{t}}B),}

 where *t**B* is the transpose of *B* (defined above). Derived quadratic form
For any bilinear form *B* : *V* × *V* → *K*, there exists an associated quadratic form *Q* : *V* → *K* defined by *Q* : *V* → *K*, *v* → *B*(*v*, *v*). When char(*K*) ≠ 2, the quadratic form *Q* is determined by the symmetric part of the bilinear form *B* and is independent of the antisymmetric part. In this case there is a one-to-one correspondence between the symmetric part of the bilinear form and the quadratic form, and it makes sense to speak of the symmetric bilinear form associated with a quadratic form. When char(*K*) = 2 and dim *V* > 1, this correspondence between quadratic forms and symmetric bilinear forms breaks down. Reflexivity and orthogonality
Definition: A bilinear form *B* : *V* × *V* → *K* is called reflexive if *B*(*v*, *w*) = 0 implies *B*(*w*, *v*) = 0 for all *v*, *w* in *V*. Definition: Let *B* : *V* × *V* → *K* be a reflexive bilinear form, *v*, *w* in *V* are orthogonal with respect to *B* if *B*(*v*, *w*) = 0. A bilinear form *B* is reflexive if and only if it is either symmetric or alternating.[4] In the absence of reflexivity we have to distinguish left and right orthogonality. In a reflexive space the left and right radicals agree and are termed the kernel or the radical of the bilinear form: the subspace of all vectors orthogonal with every other vector. A vector *v*, with matrix representation *x*, is in the radical of a bilinear form with matrix representation *A*, if and only if *Ax* = 0 ⇒ *xTA* = 0. The radical is always a subspace of *V*. It is trivial if and only if the matrix *A* is nonsingular, and thus if and only if the bilinear form is nondegenerate. Suppose *W* is a subspace. Define the orthogonal complement[5] 




W

⊥



=
{
v
|
B
(
v
,
w
)
=
0
 
for 
all
 
w
∈
W
}
.


{\displaystyle W^{\perp }=\left\{\mathbf {v} \mid B(\mathbf {v} ,\mathbf {w} )=0{\text{ for all }}\mathbf {w} \in W\right\}.}

 For a non-degenerate form on a finite-dimensional space, the map *VW* → *W* ⊥  is bijective, and the dimension of *W* ⊥  is dim(*V*) − dim(*W*). Different spaces
Much of the theory is available for a bilinear mapping from two vector spaces over the same base field to that field *B* : *V* × *V* → *K*. Here we still have induced linear mappings from *V* to *W*\*, and from *W* to *V*\*. It may happen that these mappings are isomorphisms; assuming finite dimensions, if one is an isomorphism, then the other must be. When this occurs, *B* is said to be a perfect pairing. In finite dimensions, this is equivalent to the pairing being nondegenerate (the spaces necessarily having the same dimensions). For modules (instead of vector spaces), just as how a nondegenerate form is weaker than a unimodular form, a nondegenerate pairing is a weaker notion than a perfect pairing. A pairing can be nondegenerate without being a perfect pairing, for instance 




Z

×

Z

→

Z

via
(
x
,
y
)
↦
2
x
y


{\displaystyle B(\mathbf {u} ,\mathbf {v} )=2xy}

 is nondegenerate, but induces multiplication by 2 on the map 




Z

→

Z

∗


.


{\displaystyle B(\mathbf {u} ,\mathbf {v} )\in \mathbb {Z} .}

 Generalization to modules
Given a ring *R* and a right *R*-module *M* and its dual module *M*\*, a mapping *B* : *M*\* × *M* → *R* is called a bilinear form if *B*(*u* + *v*, *x*) = *B*(*u*, *x*) + *B*(*v*, *x*) *B*(*u*, *x* + *y*) = *B*(*u*, *x*) + *B*(*u*, *y*) *B*(*u*, *x**y*) = α*B*(*u*, *x**y*) for all *u*, *v* ∈ *M*, all *x*, *y* ∈ *M* and all α, β ∈ *R*. The mapping (·, ·): *M*\* × *M* → *R* (·, *u*, *x*) ⇒ *u*(*x*) is known as the natural pairing, also called the canonical bilinear form on *M*\* × *M*.[8] A linear map *S* : *M*\* → *M*\* · *u* ⇒ *S*(*u*) induces the bilinear form *B* : *M*\* × *M* → *R* (·, *u*, *x*) ⇒ (*S*(*u*), *x*) and a linear map *T* : *M* → *M* · *x* ⇒ *T*(*x*) induces the bilinear form *B* : *M*\* × *M* → *R* (·, *u*, *x*) ⇒ (·, *T*(*x*)). Conversely, a bilinear form *B* : *M*\* × *M* → *R* induces the *R*-linear maps *S* : *M*\* → *M*\* · *u* ⇒ (x ⇒ *B*(*u*, *x*)) and *T* : *M* → *M*\* · *x* ⇒ (· ⇒ *B*(*u*, *x*)). Here, *M*\*\* denotes the double dual of *M*. See also Bilinear map Bilinear operator Inner product space Linear form Multilinear form Polar space Quadratic form Sesquilinear form System of bilinear equations Citations
^ Chapter 3. Bilinear forms — Lecture notes for MA1212 (PDF). 2021-01-16.
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